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Twist-4 photon helicity-flip amplitude in DVCS on a nucleon in the Wandzura-Wilczek approximation

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Abstract. We computed the twist-4 part of the photon spin-flip amplitude in deeply virtual Compton scattering on a nucleon in the Wandzura–Wilczek approximation. We found a factorizable contribution, which arises from photon scattering on quarks with non-zero angular momentum along the collision axis. As the genuine twist-2 amplitude arises at the NLO, for moderate virtualities of the hard photon, $Q^2 \leq 10\,\mathrm{GeV}^2$, a kinematical twist-4 correction can give a numerically important contribution to the photon helicity-flip amplitude.

Introduction

Deeply virtual Compton scattering (DVCS) [1,2] on a nucleon, $\gamma^* N \to \gamma N'$, is perhaps the cleanest hard reaction sensitive to the skewed parton distributions (SPD). For that reason in recent years DVCS has been the subject of extensive theoretical investigations. First experimental data have also recently become available (see e.g. [3–6]) and much more data are expected from JLAB, DESY, and CERN in the near future. Due to factorization theorems [7–9] the leading term in the $1/Q^2$ expansion of the DVCS amplitude, where Q^2 is the large virtuality of the hard photon, can be expressed in terms of twist-2 skewed parton distributions. However, as the typical experimentally accessible values of Q^2 are by no means large, studies of the power suppressed (higher twist) corrections to the DVCS amplitude are very important from the phenomenological point of view. The leading power corrections are of the order $1/Q$, or twist-3, and therefore they may have significant effects on some of DVCS observables. Note also that twist-3 corrections typically scale as $(-t)^{1/2}/Q$, with t denoting the square of the momentum transfer, so the size of twist-3 corrections increases with t. It follows that taking into account these corrections is mandatory for understanding continuation of the twist-2 part of the DVCS amplitude to $t = 0$.

An interesting feature of the DVCS amplitude on a nucleon is that it receives a contribution from the photon helicity-flip process, which is forbidden by the angular-momentum conservation in the forward DIS case. In the leading-twist approximation, this amplitude appears at the NLO level and, if measured, can provide unique information about the tensor gluon skewed parton distribution in a nucleon. In this case, analyzing corresponding power corrections is even more important as there is no prejudice about how large the twist-2 amplitude can be. The simplest estimate can be obtained by calculating the so-called Wandzura–Wilczek, or kinematical power corrections. From a phenomenological point of view such a calculation is crucial for the future studies of the photon helicity-flip amplitude in DVCS.

The remainder of this paper is organized as follows: in the next section we discuss general features of the DVCS amplitude on the nucleon. The following two sections are devoted to a discussion of the photon helicity-flip amplitude and to the calculation of the kinematical twist four correction, respectively. Finally, we conclude this article. Technical details of the present calculation are summarized in the appendix.

DVCS amplitude on a nucleon

Let p, p' and q, q' denote momenta of the initial and final nucleons and photons, respectively. The amplitude of the virtual Compton scattering process

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p'), \tag{1}$$

is defined in terms of the nucleon matrix element of the T-product of two electromagnetic currents:

$$T^{\mu\nu} = -\mathrm{i} \int \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}(q+q')x/2} \times \langle p'| T\left[J^{\mu}_{\mathrm{e.m.}}(x/2) J^{\nu}_{\mathrm{e.m.}}(-x/2)\right] |p\rangle, \tag{2}$$

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where Lorentz indices μ and ν correspond to the virtual, respectively the real photon.

We shall consider the Bjorken limit, where $-q^2 = Q^2 \to \infty$, $2(p \cdot q) \to \infty$, with $x_B = Q^2/2(p \cdot q)$ constant, and $t \equiv (p-p')^2 \ll Q^2$. We introduce two light-like vectors n, n^* such that

$$n \cdot n = 0, \quad n^* \cdot n^* = 0, \quad n \cdot n^* = 1. \tag{3}$$

We shall work in a reference frame where the average nucleon momenta $P = (1/2)(p+p')$ and the virtual photon momentum q are collinear along the z-axis and have opposite directions. Such a choice of the frame results in the following decomposition of the momenta [10]:

$$\begin{aligned} P &= n^* + \frac{\bar{m}^2}{2} n, \\ q &= -2\xi' n^* + \frac{Q^2}{4\xi'} n, \\ \Delta &= p' - p = -2\xi n^* + \bar{m}^2 \xi n + \Delta_\perp, \end{aligned} \tag{4}$$

with $\bar{m}^2 = m^2 - t/4$, $t = \Delta^2$ being the squared momentum transfer, and

$$2\xi = 2\xi' \frac{Q^2 - t}{Q^2 + 4\xi'^2 \bar{m}^2}. \tag{5}$$

Finally, ξ' is given by

$$\begin{aligned} \xi' &= \frac{2}{\frac{2-x_B}{x_B} + t/Q^2 + \sqrt{\left(\frac{2-x_B}{x_B} + t/Q^2\right)^2 + 16\frac{\bar{m}^2}{Q^2}}} \\ &= \frac{x_B}{2 - x_B} + O(1/Q^2), \end{aligned} \tag{6}$$

with

$$x_B = \frac{Q^2}{2p \cdot q}.$$

We define the transverse metric and antisymmetric transverse epsilon tensors[1]:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu n^{*\nu} - n^\nu n^{*\mu}, \quad \epsilon_{\mu\nu}^\perp = \epsilon_{\mu\nu\alpha\beta} n^\alpha n^{*\beta}. \tag{7}$$

In the following, we shall use the shorthand notation for

$$a^+ \equiv a_\mu n^\mu, \quad a^- \equiv a_\mu n^{*\mu}, \tag{8}$$

where a_μ is an arbitrary Lorentz vector.

In the LO approximation in the QCD coupling α_S, but including $1/Q$ corrections, the DVCS amplitude on a nucleon has the form [11,12]

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} + T_3^{\mu\nu}, \tag{9}$$

$$\begin{aligned} T_1^{\mu\nu} = -\frac{1}{2}\int_{-1}^{1} \mathrm{d}x \Bigg\{ &\left[g_\perp^{\mu\nu} + \frac{P^\nu \Delta_\perp^\mu}{(Pq)} \right] n^\rho F_\rho(x,\xi) C^+(x,\xi) \\ &- \left[g_\perp^{\nu\alpha} + \frac{P^\nu \Delta_\perp^\alpha}{(Pq)} \right] \mathrm{i}\epsilon_\alpha^{\perp\mu} n^\rho \widetilde{F}_\rho(x,\xi) C^-(x,\xi) \Bigg\}, \end{aligned} \tag{10}$$

$$\begin{aligned} T_2^{\mu\nu} = &\frac{(q+4\xi P)^\mu}{(Pq)} \left[g_\perp^{\nu\alpha} + \frac{P^\nu \Delta_\perp^\alpha}{(Pq)} \right] \\ &\times \frac{1}{2}\int_{-1}^{1} \mathrm{d}x \Big\{ F_\alpha(x,\xi) C^+(x,\xi) \\ &-\mathrm{i}\epsilon_{\alpha\rho}^\perp \widetilde{F}^\rho(x,\xi) C^-(x,\xi) \Big\}, \end{aligned} \tag{11}$$

$$\begin{aligned} T_3^{\mu_\perp\nu} = &\frac{(q+2\xi P)^\nu}{(Pq)} \frac{1}{2} \int_{-1}^{1} \mathrm{d}x \Big\{ F^{\mu_\perp}(x,\xi) C^+(x,\xi) \\ &+\mathrm{i}\epsilon_\perp^{\mu\rho} \widetilde{F}_\rho(x,\xi) C^-(x,\xi) \Big\}, \end{aligned} \tag{12}$$

where to the twist-3 accuracy

$$\begin{aligned} P &= \frac{1}{2}(p+p') = n^*, \quad \Delta = p' - p = -2\xi P + \Delta_\perp, \\ q &= -2\xi P + \frac{Q^2}{4\xi} n, \quad q' = q - \Delta = \frac{Q^2}{4\xi} n - \Delta_\perp, \end{aligned} \tag{13}$$

with ξ equal to its leading-order value $\xi = x_B/(2 - x_B)$. The leading-order coefficient functions are

$$C^\pm(x,\xi) = \frac{1}{x - \xi + \mathrm{i}\varepsilon} \pm \frac{1}{x + \xi - \mathrm{i}\varepsilon},$$

and the skewed distributions $F_\mu(x,\xi)$ and $\widetilde{F}_\mu(x,\xi)$ are defined in terms of the nonlocal light-cone quark operators[2]:

$$F_\mu(x,\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{-\mathrm{i}x\lambda} \langle p' | \bar{\psi}\left(\frac{1}{2}\lambda n\right) \gamma_\mu \psi \left(-\frac{1}{2}\lambda n\right) | p \rangle, \tag{14}$$

$$\widetilde{F}_\mu(x,\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{-\mathrm{i}x\lambda} \langle p' | \bar{\psi}\left(\frac{1}{2}\lambda n\right) \gamma_\mu \gamma_5 \psi \left(-\frac{1}{2}\lambda n\right) | p \rangle. \tag{15}$$

In the above expression for the DVCS amplitude the first term, $T_1^{\mu\nu}$, corresponds to the scattering of transversely polarized virtual photons. This part of the amplitude depends only on twist-2 SPDs H, E and $\widetilde{H}, \widetilde{E}$. The twist-3 terms proportional to[3] $P^\mu \Delta_\perp^\nu/(Pq)$ are required to ensure the proper electromagnetic gauge invariance of the amplitude:

$$q_\mu T_1^{\mu\nu} = T_1^{\mu\nu} q'_\nu = 0. \tag{16}$$

The second term $T_2^{\mu\nu}$ corresponds to the contribution of the longitudinal polarization of the virtual photon. This term depends only on the new "transverse" SPDs $F^{\mu_\perp}$ and $\widetilde{F}^{\mu_\perp}$. Defining the longitudinal polarization vector of the virtual photon by

$$\varepsilon_L^\mu(q) = \frac{1}{Q}\left(2\xi P^\mu + \frac{Q}{4\xi} n^\mu\right), \tag{17}$$

[1] The Levi-Civita tensor $\epsilon_{\mu\nu\alpha\beta}$ is defined as the totally antisymmetric tensor with $\epsilon_{0123} = 1$

[2] The gauge link between points on the light-cone is not shown but always assumed

[3] We adopt here kinematical definition of twist i.e., terms suppressed by $1/Q$ are of twist-3

one can easily calculate the DVCS amplitude for the longitudinal polarization of the virtual photon ($L \to T$ transition), which is purely twist-3:

$$\varepsilon^L_\mu T^{\mu\nu_\perp} = \frac{2\xi}{Q}\int_{-1}^{1} \mathrm{d}x (F^{\nu_\perp} C^+(x,\xi) - \mathrm{i}\varepsilon_\perp^{\nu_\perp \alpha}\widetilde{F}_\alpha C^-(x,\xi)). \tag{18}$$

The skewed parton distributions F_μ and $\widetilde{F}_\mu$ can be related to the twist-2 SPDs $H, E, \widetilde{H}$ and $\widetilde{E}$ through the so-called Wandzura–Wilczek relations [12–17]. To derive these relations one assumes that non-forward nucleon matrix elements of gauge invariant operators of the type $\bar{\psi}G\psi$, i.e. involving quark–gluon correlations, are small. The WW relations for the case of the nucleon SPDs have the form [12,15]:

$$\begin{aligned} F^{WW}_\mu(x,\xi) &= \frac{\Delta_\mu}{2\xi}\langle\!\langle \frac{1}{m}\rangle\!\rangle E(x,\xi) - \frac{\Delta_\mu}{2\xi}\langle\!\langle \gamma_+\rangle\!\rangle (H+E)(x,\xi) \\ &+ \int_{-1}^{1}\mathrm{d}u G_\mu(u,\xi) W_+(x,u,\xi) \\ &+ \mathrm{i}\epsilon_{\perp\mu\alpha}\int_{-1}^{1}\mathrm{d}u \widetilde{G}^\alpha(u,\xi) W_-(x,u,\xi), \end{aligned} \tag{19}$$

$$\begin{aligned} \widetilde{F}^{WW}_\mu(x,\xi) &= \Delta_\mu \frac{1}{2}\langle\!\langle \frac{\gamma_5}{m}\rangle\!\rangle \widetilde{E}(x,\xi) - \frac{\Delta_\mu}{2\xi}\langle\!\langle \gamma_+\gamma_5\rangle\!\rangle \widetilde{H}(x,\xi) \\ &+ \int_{-1}^{1}\mathrm{d}u \widetilde{G}_\mu(u,\xi) W_+(x,u,\xi) \\ &+ \mathrm{i}\epsilon_{\perp\mu\alpha}\int_{-1}^{1}\mathrm{d}u G^\alpha(u,\xi) W_-(x,u,\xi). \end{aligned} \tag{20}$$

Here we have introduced the shorthand notation $\langle\!\langle \ldots \rangle\!\rangle = \bar{U}(p')\ldots U(p)$, and m denotes the nucleon mass. Functions G^μ and $\widetilde{G}^\mu$ are defined by

$$\begin{aligned} G^\mu(u,\xi) &= \langle\!\langle \gamma^\mu_\perp\rangle\!\rangle (H+E)(u,\xi) \\ &+ \frac{\Delta^\mu_\perp}{2\xi}\langle\!\langle \frac{1}{m}\rangle\!\rangle \left[u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right] E(u,\xi) \\ &- \frac{\Delta^\mu_\perp}{2\xi}\langle\!\langle \gamma_+\rangle\!\rangle \left[u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right] (H+E)(u,\xi), \end{aligned} \tag{21}$$

$$\begin{aligned} \widetilde{G}^\mu(u,\xi) &= \langle\!\langle \gamma^\mu_\perp\gamma_5\rangle\!\rangle \widetilde{H}(u,\xi) \\ &+ \frac{1}{2}\Delta^\mu_\perp\langle\!\langle \frac{\gamma_5}{m}\rangle\!\rangle \left[1 + u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right] \widetilde{E}(u,\xi) \\ &- \frac{\Delta^\mu_\perp}{2\xi}\langle\!\langle \gamma_+\gamma_5\rangle\!\rangle \left[u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right] \widetilde{H}(u,\xi). \end{aligned} \tag{22}$$

The Wandzura–Wilczek kernels $W_\pm(x,u,\xi)$ have been introduced in [12,14,15]. They are defined by

$$\begin{aligned} &W_\pm(x,u,\xi) \\ &= \frac{1}{2}\left\{\theta(x>\xi)\frac{\theta(u>x)}{u-\xi} - \theta(x<\xi)\frac{\theta(u<x)}{u-\xi}\right\} \\ &\pm \frac{1}{2}\left\{\theta(x>-\xi)\frac{\theta(u>x)}{u+\xi} - \theta(x<-\xi)\frac{\theta(u<x)}{u+\xi}\right\}. \end{aligned} \tag{23}$$

The flavor dependence in the amplitude can easily be restored by a substitution:

$$F_\mu(\widetilde{F}_\mu) \to \sum_{q=u,d,s,\ldots} e_q^2 F^q_\mu(\widetilde{F}^q_\mu). \tag{24}$$

The amplitude (9) is electromagnetically gauge invariant, i.e.

$$q_\mu T^{\mu\nu} = T^{\mu\nu}(q-\Delta)_\nu = T^{\mu\nu}q'_\nu = 0, \tag{25}$$

formally to the accuracy $1/Q^2$. In order to work with an amplitude which is transverse in the sense of (25) we have kept in (11) terms of the Δ^2/Q^2 order, applying the prescription of [10,18]:

$$g^{\mu\nu}_\perp \to g^{\mu\nu}_\perp + \frac{P^\nu \Delta^\mu_\perp}{(P\cdot q')}, \tag{26}$$

for the twist-3 terms in the amplitude. Although such corrections do not form a complete set of $1/Q^2$ contributions, we prefer to work with the DVCS amplitude $T_2^{\mu\nu}$ which satisfies (25) exactly.

The last term, $T_3^{\mu\nu}$, corresponds to transverse polarization of the virtual photon. It is proportional to $(q+2\xi P) = q' + \Delta_\perp$. Contracting with the transverse polarization vector $e_\nu(q')$ of the final real photon one obtains

$$e_\nu(q')(q+2\xi P)^\nu = e_\nu(q')\Delta^\nu_\perp. \tag{27}$$

It follows that such a term does not contribute to any observable with the accuracy $O(\Delta/Q)$.

In the case of a pion target [14] it has been shown that the structure $(q+2\xi P)$ emerges as truncated to the $1/Q$ accuracy vector q'. Obviously, such a term, although formally present in the amplitude $T^{\mu\nu}$, does not contribute to any physical DVCS amplitude with the real, transverse photon in the final state. The observation that the amplitude $T_3^{\mu\nu}$ has zero projection onto physical states when the final photon is real is not unexpected. Considering the situation where both photons are virtual one finds that the amplitude $T_3^{\mu\nu}$ describes a $T \to L$ transition i.e., with incoming transverse and outgoing longitudinal photon, respectively. It therefore has to disappear in the limit when the outgoing photon is real. Indeed, the contraction $\varepsilon_{T\mu}(q) T_3^{\mu\nu}\varepsilon^*_{L\nu}(q')$ vanishes when $q'^2 \to 0$. The same situation is expected, of course, for a nucleon target.

There is another amplitude which appears at a twist-2 level, but at the α_S order. This amplitude describes DVCS of transversely polarized photons with helicity flip between the initial and final photon states, respectively. Kinematical twist-4 corrections to this amplitude are the main subject of the considerations in this paper. At the twist-2 level the helicity-flip amplitude depends on a new distribution function, the so-called tensor gluon or gluon transversity skewed parton distribution. Feynman diagrams involving photon helicity-flip contribution to DVCS have been calculated by two groups [19,20]. Their result can be represented as follows:

$$A^{\mathrm{tw2}}_{\mu\nu} = \left(\sum_f e_f^2\right)\frac{\alpha_\mathrm{s}(Q^2)}{4\pi\xi}\int_{-1}^{1}\mathrm{d}x F_{(\mu\nu)}(x,\xi) C^-(x,\xi), \tag{28}$$

where $F_{(\mu\nu)}$ is defined as a matrix element of a non-local light-cone gluon operator:

$$F_{(\mu\nu)}(x,\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} \mathrm{e}^{-\mathrm{i}x\lambda} \times \langle p'|\mathbf{S} G^a_{n\mu}\left(\frac{1}{2}\lambda n\right) G^a_{n\nu}\left(-\frac{1}{2}\lambda n\right)|p\rangle. \quad (29)$$

The symbols $\mathbf{S}$ and $(\mu\nu)$ stand for symmetrization of the two indices and removal of the trace: $\mathbf{S}O_{\mu\nu} = (1/2)O_{\mu\nu} + (1/2)O_{\nu\mu} - (1/4)g_{\mu\nu}O^\alpha_\alpha$ and $G^a_{n\nu}$ is a shorthand notation for $n^\alpha G^a_{\alpha\nu}$.

Parameterization of this matrix element in terms of independent SPDs will be discussed below. Here we note that the gluon operator (29) does not mix with quark operators and therefore the corresponding SPDs are sometimes considered to be the cleanest probes of the gluonic content of hadrons. The off-forward gluon helicity-flip SPDs in a nucleon, determined by the matrix element of a twist-2 operator (29), can be numerically as large as other gluon distributions. However, the amplitude $A^{\mathrm{tw2}}_{\mu\nu}$ arises at the NLO level i.e. it is proportional to $\alpha_{\mathrm{S}}(Q^2)$. For realistic values of Q^2 of the order of a few GeV^2 it is therefore natural to consider also power suppressed corrections to the photon helicity-flip DVCS amplitude. In this paper we provide an estimate of such higher-twist effects by explicitly calculating a corresponding Wandzura–Wilczek (WW) contribution which originates from the handbag diagram. Note that although the WW contribution is suppressed like $1/Q^2$, it appears already at the tree level.

As we shall demonstrate in the following, a ratio of twist-2 to twist-4 amplitudes behaves then like $\alpha_{\mathrm{s}}(Q^2)/\pi : m^2/Q^2$, which is not necessarily very large if Q^2 is of the order of a few GeV^2.

Recently, a similar analysis of the WW contribution has been carried out for the process $\gamma^*\gamma \to f_2(1270)$ [21]. The analog of a helicity-flip amplitude in this process is the amplitude which describes scattering of transverse photons with different helicities. It was found that the latter amplitude is rather sensitive to power corrections in the region of $Q^2 \leq 10\,\mathrm{GeV}^2$.

The WW kinematical correction is of course not the full answer, as far as power suppressed corrections are concerned. Assuming that factorizability holds for this amplitude to $1/Q^2$ accuracy, there will be additional contributions from multi-parton operators of twist four, which we have not calculated here. Note, however, that in the chiral limit helicity is conserved along the quark line. As follows, in order to account for two units of angular momentum one has to consider the emission of two additional, transverse gluons in a collinear configuration or a WW contribution to an emission of a single gluon resulting in a configuration with one unit of angular momentum carried by partons. Current phenomenology of power corrections is consistent with the conjecture that the matrix elements of such three- or four-parton operators in a nucleon are small [22]. This observation suggests that the WW power correction discussed in this paper can provide a rather accurate numerical description of higher-twist corrections to the photon helicity-flip amplitude.

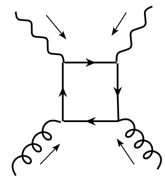

Fig. 1. Typical diagram for the photon–gluon scattering with photon helicity flip. Arrows indicate polarizations of photons and gluons, respectively

Photon helicity-flip amplitude in DVCS

As has been discussed in [23] the LO handbag diagram contribution to DVCS leads to an effective s-channel photon helicity conservation for the leading-twist amplitude. At the NLO a new twist-2 amplitude $A^{\mathrm{tw2}}_{\mu\nu}$ arises, which describes a DVCS process with photon helicity flip.

The twist-2 photon helicity-flip amplitude is absent in the handbag diagram because of conservation of the angular momentum along the photon–parton collision axis. As the photon is a vector particle, to allow for flip of its helicity one has to compensate for two units of angular momentum. For the collinear twist-2 partonic amplitude it is only possible by a simultaneous flip of gluon helicities, see Fig. 1. As quarks have spin 1/2, their helicity flip can provide at most one unit of angular momentum. As a consequence, twist-2 photon helicity-flip amplitude is sensitive to the helicity-flip gluon distribution in a nucleon.

However, a similar angular-momentum conservation argument shows that such a distribution is forbidden in the forward limit, i.e. in DIS, on a spin 1/2 target. In the off-forward case a transverse component of the momentum transfer $\Delta_\perp$ can provide one unit of angular momentum, so DVCS offers the unique opportunity to investigate the helicity-flip gluon distribution in a nucleon. As discussed in detail in [19,20,23] such information can be extracted from azimuthal asymmetries $\propto \cos 3\phi$ of the cross-section, where ϕ is the angle between the lepton and nucleon planes.

Parameterization of the twist-2 gluonic matrix element (29) was first introduced in [20] and recently revised in [24]. In the notation of [24] one has

$$\begin{aligned} F^{(\mu\nu)}(x,\xi) &= -\mathbf{S}\frac{\Delta^\mu_\perp}{4m}\left\{ H^g_T(x,\xi)\langle\!\langle \mathrm{i}\sigma^{+\nu}\rangle\!\rangle + \tilde{H}^g_T(x,\xi)\frac{\Delta^\nu_\perp}{m}\langle\!\langle 1/m\rangle\!\rangle \right. \\ &\quad + E^g_T(x,\xi)\left(\frac{\Delta^\nu_\perp}{2m}\langle\!\langle\gamma^+\rangle\!\rangle + \frac{1}{m}\langle\!\langle\gamma^\nu_\perp\rangle\!\rangle\right) \\ &\quad \left. + \tilde{E}^g_T(x,\xi)\frac{1}{m}\langle\!\langle\gamma^\nu_\perp\rangle\!\rangle \right\}. \end{aligned} \quad (30)$$

Note that μ and ν in (30) are understood as transverse Lorentz indices. As a consequence, the twist-2 helicity-flip amplitude (28) is gauge invariant to the $1/Q$ accuracy:

$$q^\mu A^{\mathrm{tw2}}_{\mu\nu} = 0, \quad A^{\mathrm{tw2}}_{\mu\nu} q'^\nu = O(1/Q). \quad (31)$$

A similar situation has been encountered before in the case of the amplitude $T_1^{\mu\nu}$. To get a fully gauge invariant result one has to include in the calculation the kinematical twist-3 contribution to the helicity-flip amplitude. As such a term arises at the NLO and is not related to the contribution from the handbag diagram which we are going to discuss here, it will not be considered further. Instead, we shall use prescription (26) in order to obtain a gauge invariant expression. As a consequence one can combine twist-2 and twist-4 contributions and write the helicity-flip amplitude $A^{\mu\nu}$ in the following form:

$$A^{\mu\nu} = \frac{1}{2}\left[g_\perp^{\mu i}\left(g_\perp^{\nu j} + \frac{P^\nu \Delta_\perp^j}{(Pq)}\right) + g_\perp^{\mu j}\left(g_\perp^{\nu i} + \frac{P^\nu \Delta_\perp^i}{(Pq)}\right) - \left(g_\perp^{\mu\nu} + \frac{P^\nu \Delta_\perp^\mu}{(Pq)}\right) g_\perp^{ij}\right]\left[A_{(ij)}^{\mathrm{tw2}} + \frac{m^2}{Q^2} A_{(ij)}^{\mathrm{tw4}}\right]. \quad (32)$$

Note that $A^{\mu\nu}$ is a general photon helicity-flip amplitude, while $A_{(ij)}^{\mathrm{tw2}}$ is the leading-twist contribution (28). The appropriate Lorentz projection operator has been written in order to make explicit the gauge invariant, symmetric and traceless form of the helicity-flip amplitude. The twist-4 part, arising from the handbag diagram through the WW mechanism can be parameterized as $(m^2/Q^2)A_{(ij)}^{\mathrm{tw4}}$, with m being the nucleon mass. A convenient physical interpretation of this term follows from the observation that beyond the leading-twist approximation one can imagine partons as carrying non-zero orbital angular momentum along the collision axis. That allows quarks to participate in the LO, i.e. through the handbag diagram, in the photon helicity-flip amplitude. As two units of angular momentum have to flow through the hard vertex, such an amplitude is suppressed by $1/Q^2$ and represents a twist-4 contribution.

Following [24], one observes that the twist-2 gluonic matrix element (29) can be parameterized in terms of four independent SPDs: $H_T^g, E_T^g, \tilde{H}_T^g$ and $\tilde{E}_T^g$ associated with four transverse tensor structures:

$$\text{chiral even:} \quad \mathbf{S}\frac{1}{m^2}\Delta_\perp^i \langle\!\langle \gamma_\perp^j \rangle\!\rangle, \quad \mathbf{S}\frac{\Delta_\perp^i \Delta_\perp^j}{m^2}\langle\!\langle \gamma^+ \rangle\!\rangle, \quad (33)$$

$$\text{chiral odd:} \quad \mathbf{S}\frac{1}{m}\Delta_\perp^i \langle\!\langle \mathrm{i}\sigma^{j+} \rangle\!\rangle, \quad \mathbf{S}\frac{\Delta_\perp^i \Delta_\perp^j}{m^2}\langle\!\langle 1/m \rangle\!\rangle. \quad (34)$$

Here, the four independent structures correspond to four independent helicity-flip amplitudes in the gluon–nucleon system. As the number of independent quark–nucleon helicity-flip amplitudes is the same[4], one expects that these tensor structures will appear in the twist-4 amplitude calculated in the WW approximation as well. It follows that the same basis of Dirac structures, (33) and (34), can be used as the basis for an expansion of $A_{(ij)}^{\mathrm{tw4}}$.

Photon helicity-flip amplitude in the Wandzura-Wilczek approximation

Let us now discuss briefly the calculation of the WW contribution to the photon helicity-flip amplitude in DVCS.

[4] Note that we consider here only amplitudes with photon helicity flip by two units

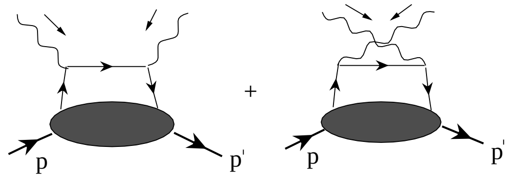

Fig. 2. Typical handbag diagrams which contribute to the twist-4 amplitude discussed in the text. Arrows indicate the polarization of the photons. Angular-momentum conservation requires that quarks carry an orbital angular momentum along the collision axis

Formally, the WW contribution arises because operators with external derivatives with respect to a total translation in a transverse direction give a non-zero contribution in the DVCS kinematics.

The photon helicity-flip amplitude (32) is symmetric in the indices μ and ν and therefore it arises from the symmetric part of the T-product of the electromagnetic currents in (2). The tree-level contribution results from the handbag diagram depicted in Fig. 2. It can be written as

$$T^{\mu\nu} = \frac{1}{\pi^2}\int \mathrm{d}^4 x \mathrm{e}^{-\mathrm{i}(q+q')x} s_{\lambda\sigma}^{\mu\nu}\frac{x_\lambda}{x^4} \times \langle p'|\bar{\psi}(x)\gamma_\sigma\psi(-x) - \bar{\psi}(-x)\gamma_\sigma\psi(x)|p\rangle + \dots \quad (35)$$

where the ellipses denote the contribution antisymmetric in μ, ν, and $s_{\lambda\sigma}^{\mu\nu} = g_\lambda^\mu g_\sigma^\nu + g_\sigma^\mu g_\lambda^\nu - g^{\mu\nu} g_{\lambda\sigma}$. In order to obtain the twist-4 WW contribution to the amplitude (32), we have extracted from the matrix element

$$\langle p'|\bar{\psi}(x)\gamma_\sigma\psi(-x)|p\rangle \quad (36)$$

terms linear and bilinear in transverse structures $\gamma_\perp$ and $\Delta_\perp$. The linear terms can in principle contribute because to obtain the full answer one has to expand the exponent $\mathrm{e}^{-\mathrm{i}(q+q')x}$ in $\Delta_\perp$ as well. In the WW approximation one neglects, as usual, contribution from quark–gluon operators. The final answer is then given in terms of twist-2 SPDs related to the matrix elements of vector and axial operators H, E and $\widetilde{H}, \widetilde{E}$ respectively; see (19) and (20).

The calculation is straightforward and technically rather close to the approach described in [15]. Technical details of the present work are summarized in the appendix. For a detailed discussion of a similar calculation, but with matrix elements parameterized in terms of double distributions, see [14]. Note that because in the WW approximation one neglects quark–gluon operators, gluon emission diagrams, calculated in [12,25], have not been taken into account here.

We have also found that apart from terms which contribute to the photon helicity-flip amplitude (37) in the WW approximation, one finds also a singular, *non-factorizable* term $\sim \Delta_\perp^\mu \Delta_\perp^\nu/(Pq)$ which contributes to the amplitude $T_3^{\mu\nu}$ in such a way that the factor $(q + 2\xi P)^\nu$ there becomes equal to q'^ν. Thus, the non-factorizable term gives no contribution to the physical DVCS amplitudes with a real photon in the final state, as discussed in the previous section.

The explicit expression for the twist-4 amplitude defined in (32) reads:

$$
\begin{aligned}
A^{(ij)}_{\rm tw4} &= \xi \int_{-1}^{1} {\rm d}t C^{-}(t,\xi) \\
&\times \Bigg\{ \frac{1}{m^2}\left(\Delta^i_\perp G^j(u,\xi) + \Delta^j_\perp G^i(u,\xi) - \text{trace}\right) \\
&\quad \otimes U_-(u,t,\xi) \\
&+ \frac{1}{m^2}\left(\Delta^i_\perp {\rm i}\epsilon^{jk}_\perp \tilde G_k(u,\xi) + \Delta^j_\perp \epsilon^{ik}_\perp \tilde G_k(u,\xi) - \text{trace}\right) \\
&\quad \otimes U_+(u,t,\xi) \Bigg\} \qquad (37) \\
&+ 2\xi \int_{-1}^{1} {\rm d}t \left[\ln(t-\xi+{\rm i}\epsilon) - \ln(t+\xi-{\rm i}\epsilon)\right] \\
&\times \Bigg\{ H^{ij}_{\rm S}(u,\xi) + G^{ij}_{\rm S}(u,\xi) \otimes W_+(u,t,\xi) \\
&+ \frac{1}{2}\left({\rm i}\epsilon^{ik}_\perp \tilde G^j_k + {\rm i}\epsilon^{jk}_\perp \tilde G^i_k\right)(u,\xi) \otimes W_-(u,t,\xi) - \text{traces} \Bigg\}.
\end{aligned}
$$

This is the main result of the present paper. For the sake of clarity, we have introduced a shorthand notation for the convolution integrals, e.g.

$$
G_k(u,\xi) \otimes U_+(u,t,\xi) \equiv \int_{-1}^{1} {\rm d}u G_k(u,\xi) U_+(u,t,\xi). \qquad (38)
$$

The flavor structure can be restored according to (24). Note that although the twist-2 amplitude (28) is a flavor singlet, the twist-4 WW correction arising from the handbag diagram is a mixture of singlet and non-singlet flavor contributions. All tensors which appear above are understood to have transverse Lorentz indices. The functions $G_k, \tilde G_k$ and kernels $W_\pm(u,t,\xi)$ are defined in (19) and their explicit form is given in (21), (22) and (23). The functions $H^{ij}_S, G^{ij}_S, \tilde G^{ij}$ and the kernels $U_\pm(u,t,\xi)$ are defined below.

Explicit expressions for $U_\pm(u,t,\xi)$ read

$$
\begin{aligned}
U_\pm(u,x,\xi) &= \frac{1}{2}(x-\xi) \\
&\times \left\{ \theta(x>\xi)\frac{\theta(u>x)}{(u-\xi)^2} - \theta(x<\xi)\frac{\theta(u<x)}{(u-\xi)^2} \right\} \pm \frac{1}{2}(x+\xi) \\
&\times \left\{ \theta(x>-\xi)\frac{\theta(u>x)}{(u+\xi)^2} - \theta(x<-\xi)\frac{\theta(u<x)}{(u+\xi)^2} \right\}. \qquad (39)
\end{aligned}
$$

To establish factorization, it is crucial to inspect the properties of the convolution integrals which appear in (37) at the points $t = \pm\xi$. For a test function $f(u,\xi)$ one readily finds

$$
\begin{aligned}
&\lim_{\varepsilon\to 0} f(u,\xi) \otimes \Big[U_-(u, t=\pm\xi+\varepsilon, \xi) \\
&\quad - U_-(u, t=\pm\xi-\varepsilon, \xi) \Big] = 0, \\
&\lim_{\varepsilon\to 0} f(u,\xi) \otimes \Big[U_+(u, t=\pm\xi+\varepsilon, \xi) \\
&\quad - U_+(u, t=\pm\xi-\varepsilon, \xi) \Big] \\
&\quad = f(\pm\xi+0,\xi) - f(\pm\xi-0,\xi). \qquad (40)
\end{aligned}
$$

One observes that the kernel U_+ appears in (37) in convolution with the function $\tilde G_k(u,\xi)$ which is defined in terms of twist-2 SPDs and their derivatives, see (22). Note that although in general the derivatives of a SPD with respect to x and ξ are discontinuous at $x = \pm\xi$, the combinations $(x\partial_x + \xi\partial_\xi)\widetilde E(\widetilde H)(x,\xi)$ are continuous at these points [15]. Hence, from (22) it follows that $\tilde G_k(u,\xi)$ has no discontinuities at $u = \pm\xi$. As a result, the convolution integrals which appear in the first two terms in (37) define a continuous function of t at $t = \pm\xi$ and factorization is not violated.

The third term in (37) has only logarithmic, integrable singularities at the points $x = \pm\xi$ in the coefficient function. One concludes therefore that this convolution integral is also well defined. Explicit expressions for the functions $H^{ij}_{\rm S}, G^{ij}_{\rm S}, \tilde G^{ij}$ read

$$
\begin{aligned}
H^{ij}_{\rm S}(u,\xi) &= \frac{\Delta^i_\perp \Delta^j_\perp}{4m^2\xi^2}\left(1 - \xi\frac{\partial}{\partial\xi}\right) \\
&\times \left\{ \langle\!\langle \frac{1}{m} \rangle\!\rangle E(u,\xi) - \langle\!\langle \gamma^+ \rangle\!\rangle (H+E)(u,\xi) \right\} \\
&- \frac{1}{2m}\left\{ \frac{\Delta^i_\perp}{2m\xi}\langle\!\langle \gamma^j_\perp \rangle\!\rangle + \frac{\Delta^j_\perp}{2m\xi}\langle\!\langle \gamma^i_\perp \rangle\!\rangle \right\}(H+E)(u,\xi), \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
G^{ij}_{\rm S}(u,\xi) &= \langle\!\langle \gamma^+ \rangle\!\rangle \frac{\Delta^i_\perp \Delta^j_\perp}{4m^2\xi^2} \\
&\times \left[\xi^2\frac{\partial^2}{\partial\xi^2} - \left(1-\xi\frac{\partial}{\partial\xi}\right) u\frac{\partial}{\partial u} \right](H+E)(u,\xi) \\
&- \langle\!\langle \frac{1}{m} \rangle\!\rangle \frac{\Delta^i_\perp \Delta^j_\perp}{4m^2\xi^2}\left[\xi^2\frac{\partial^2}{\partial\xi^2} - \left(1-\xi\frac{\partial}{\partial\xi}\right) u\frac{\partial}{\partial u} \right] E(u,\xi) \\
&- \frac{1}{2m}\left\{ \frac{\Delta^i_\perp}{2m\xi}\langle\!\langle \gamma^j_\perp \rangle\!\rangle + \frac{\Delta^j_\perp}{2m\xi}\langle\!\langle \gamma^i_\perp \rangle\!\rangle \right\}\left[2\xi\frac{\partial}{\partial\xi} + u\frac{\partial}{\partial u} \right] \\
&\times (H+E)(u,\xi), \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
\tilde G^{ij}(u,\xi) &= \langle\!\langle \gamma^+\gamma_5 \rangle\!\rangle \frac{\Delta^i_\perp \Delta^j_\perp}{4m^2\xi^2} \\
&\times \left[\xi^2\frac{\partial^2}{\partial\xi^2} - u\frac{\partial}{\partial u}\left(1-\xi\frac{\partial}{\partial\xi}\right) \right] \widetilde H(u,\xi) \\
&- \langle\!\langle \frac{\gamma_5}{m} \rangle\!\rangle \frac{\Delta^i_\perp \Delta^j_\perp}{4m^2\xi}\left[\xi^2\frac{\partial^2}{\partial\xi^2} + \xi\frac{\partial}{\partial\xi}\left(2 + u\frac{\partial}{\partial u}\right) \right] \tilde E(u,\xi) \\
&- \frac{1}{m}\left\{ \frac{\Delta^i_\perp}{2m\xi}\langle\!\langle \gamma^j_\perp\gamma_5 \rangle\!\rangle + \frac{\Delta^j_\perp}{2m\xi}\langle\!\langle \gamma^i_\perp\gamma_5 \rangle\!\rangle \right\} \xi\frac{\partial}{\partial\xi}\widetilde H(u,\xi) \\
&+ \frac{1}{m}\left\{ \frac{\Delta^j_\perp}{2m\xi}\langle\!\langle \gamma^i_\perp\gamma_5 \rangle\!\rangle - \frac{\Delta^i_\perp}{2m\xi}\langle\!\langle \gamma^j_\perp\gamma_5 \rangle\!\rangle (1 + u\frac{\partial}{\partial u}) \right\} \\
&\times \widetilde H(u,\xi). \qquad (43)
\end{aligned}
$$

Note that the functions $\tilde G_k$ and $\tilde G^{ij}$ contain Dirac structures which are defined with the help of the γ_5 matrix. Using relations which follow from the Gordon identities one can express them through the basic structures (33) and (34). For example, one finds

$$\begin{aligned}
&\epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma_5\rangle\!\rangle \\
&\quad = -\frac{1}{2}t\langle\!\langle\sigma^{+}_{i}\rangle\!\rangle - 2\mathrm{i}\xi m\langle\!\langle\gamma_{\perp i}\rangle\!\rangle - \frac{1}{2}\mathrm{i}\Delta_{\perp i}(2m\langle\!\langle\gamma^{+}\rangle\!\rangle - 2\langle\!\langle 1\rangle\!\rangle), \\
&2m\epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma^{+}\gamma_5\rangle\!\rangle \\
&\quad = -2\xi\epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma_5\rangle\!\rangle - 2\mathrm{i}(2m\langle\!\langle\gamma_{\perp i}\rangle\!\rangle \\
&\qquad -\xi\Delta_{\perp i}\langle\!\langle 1\rangle\!\rangle + 2\mathrm{i}\xi\bar{m}^{2}\langle\!\langle\sigma^{+}_{i}\rangle\!\rangle), \\
&2m\epsilon^{\perp}_{ik}\langle\!\langle\gamma^{k}_{\perp}\gamma_5\rangle\!\rangle \\
&\quad = \epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma_5\rangle\!\rangle - \mathrm{i}\Delta_{\perp i}\langle\!\langle 1\rangle\!\rangle - 2\bar{m}^{2}\langle\!\langle\sigma^{+}_{i}\rangle\!\rangle, \\
&2m\epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma_{\perp j}\gamma_5\rangle\!\rangle \\
&\quad = \Delta_{\perp j}\epsilon^{\perp}_{ik}\Delta^{k}_{\perp}\langle\!\langle\gamma_5\rangle\!\rangle - \mathrm{i}\Delta_{\perp i}\Delta_{\perp j}\langle\!\langle 1\rangle\!\rangle \\
&\qquad -2\bar{m}^{2}\langle\!\langle\sigma^{+}_{i}\rangle\!\rangle + \ldots
\end{aligned} \tag{44}$$

Here the ellipses stand for terms which are proportional to δ_{ij} and therefore do not contribute to the traceless combination which enters amplitude (37). Note that, as expected, all tensor structures present in the twist-2 matrix element (30) appear also in the twist-4 WW contribution.

It is important to stress that all integrals which define twist-4 amplitude $(m^2/Q^2)A^{\sigma j}_{\mathrm{tw4}}$ in (37) are well defined and therefore factorization is not violated, as far as the tree-level WW contribution to the twist-4 photon helicity-flip amplitude is concerned. This is an interesting result, as current QCD factorization theorems for exclusive processes guarantee factorization of the leading-twist contribution to DVCS only.

Let us now consider in more detail contributions of the so-called *pion pole* and *D-terms*. At small t the skewed parton distribution $\widetilde{E}$ is dominated by the chiral contribution of the pion pole [26, 29] of the form

$$\widetilde{E}^{\text{pion pole}}(x,\xi) = \frac{4g_A^2 m^2}{-t+m_\pi^2}\frac{1}{\xi}\varphi_\pi\left(\frac{x}{\xi}\right)\theta(|x|\le\xi), \tag{45}$$

where g_A is the axial charge of the nucleon and $\varphi_\pi(u)$ is the pion distribution amplitude. As was noted in [15] this contribution cancels in $\tilde{G}^k$. It is easy to see that in the function $\tilde{G}^{ij}$ defined in (43) the pion pole contribution again vanishes under the action of the differential operator. It can be understood as a consequence of P-invariance since a pseudoscalar t-channel exchange cannot contribute to the photon helicity-flip amplitude. Vanishing of the pion pole contribution is therefore a non-trivial check of our calculation.

D-terms [27] complete parameterizations of SPDs in terms of double distributions [28]. They have the form

$$\begin{aligned}
H^{D\text{-term}}(x,\xi) &= D\left(\frac{x}{\xi}\right)\theta(|x|\le\xi), \\
E^{D\text{-term}}(x,\xi) &= -D\left(\frac{x}{\xi}\right)\theta(|x|\le\xi).
\end{aligned} \tag{46}$$

Here $D(u)$ is an odd function of its argument. Estimates in the framework of the chiral quark-soliton model of a nucleon suggest that D-term can be numerically large [30]. Calculation of the DVCS cross-section shows that the effects of the D-term can be clearly seen [15, 29]. In the present case one readily finds that D-term gives a non-zero contribution only through the function $H^{ij}_{\mathrm{S}}(u,\xi)$:

$$\begin{aligned}
H^{ij}_{\mathrm{S}}(u,\xi)|_{D\text{-term}} = &-\frac{\Delta^i_\perp\Delta^j_\perp}{4m^2\xi^2}\langle\!\langle\frac{1}{m}\rangle\!\rangle\theta(|x|\le\xi) \\
&\times\left[D\left(\frac{x}{\xi}\right) + \frac{x}{\xi}D'\left(\frac{x}{\xi}\right)\right],
\end{aligned} \tag{47}$$

where $D'(x) \equiv \mathrm{d}/\mathrm{d}x D(x)$. In all other functions introduced here the D-term contribution vanishes under the action of the differential operators.

A detailed investigation of the relative importance of the twist-4 correction as compared to the twist-2 amplitude requires models for both gluon transversity and twist-2 skewed quarks distributions in a nucleon. However, a qualitative analysis shows that the WW contribution to the photon helicity-flip amplitude can be large and its consideration is perhaps mandatory for any attempt to extract an estimate of the gluon transversity distribution from the data. Assuming that the ratio of convolution integrals of SPD with the corresponding Wilson coefficients is of the order of one, from (30) and (37) one finds that the ratio of twist-2 to twist-4 amplitudes behaves like $\alpha_{\mathrm{s}}(Q^2)/\pi : m^2/Q^2$, which gives numerically ~ 0.25 and ~ 1.2 for $Q^2 = 2$ and $10\,\mathrm{GeV}^2$, respectively. This suggests that it might be necessary to take into account the WW contribution to the photon helicity-flip amplitude up to values of Q^2 of the order of $10\,\mathrm{GeV}^2$.

Summary and conclusions

It has been recognized for some time that photon helicity-flip amplitude in DVCS on a nucleon provides a unique opportunity to study the twist-2 gluon transversity distribution in a nucleon. Because of its importance for studies of novel aspects of nucleon structure, it is mandatory to consider not only the leading-twist contribution, but the power suppressed corrections to this amplitude as well.

In this paper we have calculated the twist-4 correction to the photon helicity-flip amplitude in DVCS in the Wandzura–Wilczek approximation. It originates at the LO from scattering of the virtual photon on quarks which carry a non-zero projection of angular momentum along the collision axis. We found a factorizable formula which allows one to calculate the kinematical power correction in terms of twist-2 quark skewed parton distributions.

Numerically, the power correction discussed in this paper is relatively enhanced as compared to the twist-2 amplitude which arises at the NLO. As a consequence, for moderate virtualities of the hard photon, $Q^2 \le 10\,\mathrm{GeV}^2$, the kinematical twist-4 correction might give an important contribution to the photon helicity-flip amplitude.

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Appendix

Twist-4 contribution to the matrix element of the vector quark operator

In this appendix we present details of the calculation of the matrix element (36) in the WW approximation. In order to compute the twist-4 correction to the photon helicity-flip amplitude in DVCS one has to expand (36) to twist-4 accuracy, retaining terms which give a non-zero contribution to the symmetric, traceless amplitude $A^{\mu\nu}$. The resulting expansion for arbitrary $x^2 \neq 0$ has the form

$$\begin{aligned}\langle p'|\bar{\psi}(x)\gamma^\sigma\psi(-x)|p\rangle &= P^\sigma V_{\mathrm{tw2}}(Px,\xi) + V^{\sigma_\perp}_{\mathrm{tw3}}(Px,\xi) \\ &\quad + V^{\sigma_\perp\rho_\perp}_{\mathrm{tw4}}(Px,\xi)x_{\rho_\perp} + \dots \end{aligned} \tag{48}$$

Note that $\xi = -(1/2)(\Delta n/Pn)$ and ellipses denote the twist-4 part which results in a term $\sim g^{\mu\nu}_\perp$ and therefore do not contribute to the amplitude $A^{\mu\nu}$.

The twist-3 term $V^{\sigma_\perp}_{\mathrm{tw3}}$ in the expansion has been obtained in [12,15]; see (19). The calculation of the twist-4 term $V^{\sigma_\perp\rho_\perp}_{\mathrm{tw4}}$ is equivalent to an expansion of the matrix element (36) up to terms bilinear in the transverse structures $\gamma_\perp$ and $\Delta_\perp$. The result reads

$$\begin{aligned}
&V^{\sigma\rho}(Px,\xi)x_\rho \\
&= \mathrm{i}(x\Delta_\perp)\int_{-1}^{1}\mathrm{d}t\mathrm{e}^{2\mathrm{i}t(Px)}\Big[G^\sigma(u,\xi)\otimes U_-(u,t,\xi) \\
&\quad +\mathrm{i}\epsilon^{\sigma\rho}_\perp\tilde{G}_\rho(u,\xi)\otimes U_+(u,t,\xi)\Big] \\
&\quad -2m^2x_\rho\Bigg\{\int_{-1}^{t}\mathrm{d}u\,[H^{\sigma\rho}(u,\xi)+G^{\sigma\rho}(u,\xi)] \\
&\quad +G^{\sigma\rho}(u,\xi)\otimes[tW_+(u,t,\xi)-\xi W_-(u,t,\xi)] \\
&\quad +\mathrm{i}\epsilon^{\sigma\alpha}_\perp\tilde{G}^\rho_\alpha(u,\xi)\otimes[tW_-(u,t,\xi)-\xi W_+(u,t,\xi)]\Bigg\} \\
&\quad -\mathrm{i}(x\Delta_\perp)\int_{-1}^{1}\mathrm{d}t\mathrm{e}^{2\mathrm{i}t(Px)}\Big[G^\sigma(u,\xi)\otimes W_-(u,t,\xi) \\
&\quad +\mathrm{i}\epsilon^{\sigma\alpha}_\perp\tilde{G}_\alpha(u,\xi)\otimes W_+(u,t,\xi)\Big].
\end{aligned} \tag{49}$$

Here all Lorentz indices are understood to be transverse. The first three lines of the above expression contribute to the photon helicity-flip amplitude. The last line results in a divergent, *non-factorizable* contribution to the amplitude $T^{\mu\nu}_3$. As discussed in the text, this contribution vanishes when contracted with a polarization vector of the final, transverse, real photon.

All functions appearing in the right-hand side of the above equation, except $G^{\sigma\rho}(u,\xi)$ and $H^{\sigma\rho}(u,\xi)$, have been defined in the main body of the paper; see (19)–(22) for $G_k(u,\xi), \tilde{G}_k(u,\xi)$ and $W_\pm(u,t,\xi)$ and (39) and (43) for $U_\pm(u,t,\xi)$ and $\tilde{G}^{k\rho}(u,\xi)$, respectively. The remaining terms are defined as follows:

$$\begin{aligned}
G^{\sigma\rho}(u,\xi) &= \langle\!\langle\gamma_+\rangle\!\rangle\frac{\Delta^\sigma_\perp\Delta^\rho_\perp}{4m^2\xi^2} \\
&\quad\times\left[\xi^2\frac{\partial^2}{\partial\xi^2}-\left(1-\xi\frac{\partial}{\partial\xi}\right)u\frac{\partial}{\partial u}\right](H+E)(u,\xi) \\
&\quad -\langle\!\langle\frac{1}{m}\rangle\!\rangle\frac{\Delta^\sigma_\perp\Delta^\rho_\perp}{4m^2\xi^2}\left[\xi^2\frac{\partial^2}{\partial\xi^2}-\left(1-\xi\frac{\partial}{\partial\xi}\right)u\frac{\partial}{\partial u}\right]E(u,\xi) \\
&\quad -\frac{1}{m}\left\{\frac{\Delta^\sigma_\perp}{2m\xi}\langle\!\langle\gamma^\rho_\perp\rangle\!\rangle+\frac{\Delta^\rho_\perp}{2m\xi}\langle\!\langle\gamma^\sigma_\perp\rangle\!\rangle\right\}\xi\frac{\partial}{\partial\xi}(H+E)(u,\xi) \\
&\quad -\frac{\Delta^\sigma_\perp}{2m^2\xi}\langle\!\langle\gamma^\rho_\perp\rangle\!\rangle u\frac{\partial}{\partial u}(H+E)(u,\xi),
\end{aligned} \tag{50}$$

$$\begin{aligned}
H^{\sigma\rho}(u,\xi) &= \frac{\Delta^\sigma_\perp\Delta^\rho_\perp}{4m^2\xi^2} \\
&\quad\times\left(1-\xi\frac{\partial}{\partial\xi}\right)\left\{\langle\!\langle\frac{1}{m}\rangle\!\rangle E(u,\xi)-\langle\!\langle\gamma_+\rangle\!\rangle(H+E)(u,\xi)\right\} \\
&\quad -\frac{\Delta^\sigma_\perp}{2m^2\xi}\langle\!\langle\gamma^\rho_\perp\rangle\!\rangle(H+E)(u,\xi).
\end{aligned} \tag{51}$$

Note that, by comparing with (41) and (42), one finds

$$H^{\sigma\rho}_S(u,\xi) = \frac{1}{2}\left(H^{\sigma\rho}(u,\xi)+H^{\rho\sigma}(u,\xi)\right), \tag{52}$$

$$G^{\sigma\rho}_S(u,\xi) = \frac{1}{2}\left(G^{\sigma\rho}(u,\xi)+G^{\rho\sigma}(u,\xi)\right). \tag{53}$$

Note that one can obtain a sum rule for the twist-4 part defined in (49). To this end, let us consider the parameterization of the matrix element of the quark part of the energy momentum tensor [32]:

$$\begin{aligned}
&\frac{1}{2}\langle p'|\bar{\psi}\frac{1}{2}\left[\gamma^\mu\mathrm{i}\overleftrightarrow{D}^\nu+\gamma^\nu\mathrm{i}\overleftrightarrow{D}^\mu\right]\psi|p\rangle \\
&= [A(\Delta^2)+B(\Delta^2)]P^{\{\mu}\langle\!\langle\gamma^{\nu\}}\rangle\!\rangle \\
&\quad -P^\mu P^\nu B(\Delta^2)\langle\!\langle\frac{1}{m}\rangle\!\rangle+C(\Delta^2)(\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2)\langle\!\langle\frac{1}{m}\rangle\!\rangle \\
&\quad +\bar{C}(\Delta^2)g^{\mu\nu}m\langle\!\langle 1\rangle\!\rangle.
\end{aligned} \tag{54}$$

Here $\{\mu\nu\}$ denotes symmetrization with respect to the indices μ and ν. Contracting both sides of this equation with $n_\mu n_\nu$ and using (19) one finds [32]

$$\begin{aligned}
\int_{-1}^{1}\mathrm{d}t\,tH(t,\xi) &= B(\Delta^2)+4\xi^2C(\Delta^2), \\
\int_{-1}^{1}\mathrm{d}t\,tE(t,\xi) &= B(\Delta^2)-4\xi^2C(\Delta^2).
\end{aligned} \tag{55}$$

Note that the form factor $C(\Delta^2)$ corresponds entirely to the contribution of the D-term [27]. On the other hand, by taking the transverse, traceless projection one finds with the help of (49)

$$\frac{1}{2}\langle p'|\bar{\psi}\frac{1}{2}\left[\gamma^\mu_\perp\mathrm{i}\overleftrightarrow{D}^\nu_\perp+\gamma^\nu_\perp\mathrm{i}\overleftrightarrow{D}^\mu_\perp-\mathrm{trace}\right]\psi|p\rangle$$

$$= \frac{1}{4}\left[V^{\mu_\perp\nu_\perp}(0,\xi) + V^{\nu_\perp\mu_\perp}(0,\xi) - \mathrm{trace}\right]$$
$$= C(\Delta^2)\Delta_\perp^{(\mu}\Delta_\perp^{\nu)}\langle\!\langle\frac{1}{m}\rangle\!\rangle. \quad (56)$$

This sum rule provides a non-trivial check of the expansion (49). Direct calculation gives

$$\frac{1}{4}\left[V^{\mu_\perp\nu_\perp}(0,\xi) + V^{\nu_\perp\mu_\perp}(0,\xi) - \mathrm{trace}\right]$$
$$= (-1)\Delta_\perp^{(\mu}\Delta_\perp^{\nu)}\langle\!\langle\frac{1}{m}\rangle\!\rangle\frac{1}{8}\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\int_{-1}^{1}\mathrm{d}t tE(t,\xi). \quad (57)$$

Using (55) one finds agreement between (56) and (57).

Finally, for the sake of completeness, let us mention two useful relations which allow one to simplify the r.h.s. of (49). One can easily check that the following equations hold:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1}\mathrm{d}u\tilde{G}^{\sigma\rho}(u,\xi)\left[tW_-(u,t,\xi) - \xi W_+(u,t,\xi)\right]$$
$$= \int_{-1}^{1}\mathrm{d}u\tilde{G}^{\sigma\rho}(u,\xi)W_-(u,t,\xi),$$
$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{-1}^{1}\mathrm{d}uG^{\sigma\rho}(u,\xi)\left[tW_+(u,t,\xi) - \xi W_-(u,t,\xi)\right]$$
$$= \int_{-1}^{1}\mathrm{d}uG^{\sigma\rho}(u,\xi)W_+(u,t,\xi) - G^{\sigma\rho}(t,\xi).$$

We have made use of these relations in order to bring the expression for the amplitude (37) to the form quoted in this paper.

References

1. D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Horejsi, Fortschr. Phys. **42**, 101 (1994)
2. X. Ji, Phys. Rev. D **55**, 7114 (1997)
3. P.R. Saull [ZEUS Collaboration], hep-ex/0003030
4. Rainer Stamen [H1-Collaboration], Measurement of the Deeply Virtual Compton Scattering at Hera, H1prelim-00-17, DIS 2000 and IHEP 2000
5. A. Airapetian [HERMES Collaboration], hep-ex/0106068
6. M. Amarian [HERMES collaboration], DVCS and Exclusive Meson Production Measured by HERMES, talk at workshop Skewed Parton Distributions and Lepton–Nucleon Scattering, DESY, September 2000, http://hermes.desy.de/workshop/TALKS/talks.html
7. A.V. Radyushkin, Phys. Rev. D **56**, 5524 (1997)
8. X. Ji, J. Osborne, Phys. Rev. D **58**, 094018 (1998)
9. J.C. Collins, A. Freund, Phys. Rev. D **59**, 074009 (1999)
10. P.A. Guichon, M. Vanderhaeghen, Prog. Part. Nucl. Phys. **41**, 125 (1998) [hep-ph/9806305]
11. M. Penttinen, M.V. Polyakov, A.G. Shuvaev, M. Strikman, Phys. Lett. B **491**, 96 (2000) [hep-ph/0006321]
12. A.V. Belitsky, D. Muller, Nucl. Phys. B **589**, 611 (2000) [hep-ph/0007031]
13. N. Kivel, M.V. Polyakov, A. Schafer, O.V. Teryaev, Phys. Lett. B **497**, 73 (2001) [hep-ph/0007315]
14. A.V. Radyushkin, C. Weiss, Phys. Lett. B **493**, 332 (2000) [hep-ph/0008214]; A.V. Radyushkin, C. Weiss, Phys. Rev. D **63**, 114012 (2001) [hep-ph/0010296]
15. N. Kivel, M.V. Polyakov, Nucl. Phys. B **600**, 334 (2001) [hep-ph/0010150]
16. I.V. Anikin, O.V. Teryaev, Phys. Lett. B **509**, 95 (2001) [hep-ph/0102209]
17. A.V. Radyushkin, C. Weiss, hep-ph/0106059
18. M. Vanderhaeghen, P.A.M. Guichon, M. Guidal Phys. Rev. Lett. **80**, 5064 (1998)
19. A.V. Belitsky, D. Muller, Phys. Lett. B **486**, 369 (2000) [hep-ph/0005028]
20. P. Hoodbhoy, X. Ji, Phys. Rev. D **58**, 054006 (1998) [hep-ph/9801369]
21. V.M. Braun, N. Kivel, Phys. Lett. B **501**, 48 (2001) [hep-ph/0012220]
22. G.S. Mitchell [E155 Collaboration], hep-ex/9903055; P. Bosted [E155x Collaboration], Nucl. Phys. A **663**, 297 (2000)
23. M. Diehl, T. Gousset, B. Pire, J.P. Ralston, Phys. Lett. B **411**, 193 (1997) [hep-ph/9706344]
24. M. Diehl, Eur. Phys. J. C **19**, 485 (2001) [hep-ph/0101335]
25. I.V. Anikin, B. Pire, O.V. Teryaev, Phys. Rev. D **62**, 071501 (2000) [hep-ph/0003203]
26. L.L. Frankfurt, M.V. Polyakov, M. Strikman, hep-ph/9808449; L. Mankiewicz, G. Piller, A. Radyushkin, Eur. Phys. J. C **10**, 307 (1999) [hep-ph/9812467]; L.L. Frankfurt, P.V. Pobylitsa, M.V. Polyakov, M. Strikman, Phys. Rev. D **60**, 014010 (1999) [hep-ph/9901429]; M. Penttinen, M.V. Polyakov, K. Goeke, Phys. Rev. D **62**, 014024 (2000) [hep-ph/9909489]
27. M.V. Polyakov, C. Weiss, Phys. Rev. D **60**, 114017 (1999)
28. A.V. Radyushkin, Phys. Rev. D **59**, 014030 (1999) [hep-ph/9805342]
29. K. Goeke, M.V. Polyakov, M. Vanderhaeghen, hep-ph/0106012
30. V.Y. Petrov, P.V. Pobylitsa, M.V. Polyakov, I. Bornig, K. Goeke, C. Weiss, Phys. Rev. D **57**, 4325 (1998) [hep-ph/9710270]
31. N. Kivel, M.V. Polyakov, M. Vanderhaeghen, Phys. Rev. D **63**, 114014 (2001) [hep-ph/0012136]
32. X. Ji, Phys. Rev. Lett. **78**, 610 (1997) [hep-ph/9603249]